Fun

Econ 3030 Fall 2025

Last Lecture

Outline

1. Final Exam
2. Fun with Luca's Research Interest
   1. Completeness is crazy
   2. Incomplete Preferences and Ambiguity
   3. Uncertainty and Financial Markets

Final Exam

- Next Monday, 8 December, from 9:30am to 12:30am in WWPH 4940
- Open books and notes (including Problem Sets)
  - no communication with others or internet searches
- Do not stress out

Plan for Today

- What are the consequences of relaxing the assumption that preferences must be complete.
- There are (at least) three reasons to do so:
  - completeness is descriptively silly,
  - without completeness our models may yield interesting equilibria one would not get otherwise, and
  - relaxing assumptions is what economic theorists do and is fun.
- I will focus on incomplete preferences in a stochastic environment, and connect incompleteness to "Knightian" uncertainty or ambiguity.
  - There are other models, but this is just one lecture.

Against Completeness

Completeness does not make sense

- When preferences are complete one can rank any pair of alternatives.
  - there can be many alternatives one does not know how to rank, particularly when consumption bundles are complex.
- Things are even worse with uncertainty, as one may not know how to compare outcomes contingent on events of unknown probability.
- There is a price to pay to get rid of completeness: without it there is no function that "represents" preferences.
  - All I learned in graduate school (maximize this subject to that) could be useless.
- Some people say completeness holds because one MUST choose.
  - This confuses "choice" with "preference"
  - If one is forced to choose when alternatives cannot be ranked that choice loses part of its meaning
    - For example, revealed preference arguments change without completeness.

Lack of Completeness vs. Indifference

Definition

A preference relation $\succsim$ on X is:

- complete if for all $\mathbf{x}, \mathbf{y} \in X$, $\mathbf{x} \succsim \mathbf{y}$ or $\mathbf{y} \succsim \mathbf{x}$ or both.

- Without completeness there exist $\mathbf{x}, \mathbf{y} \in X$ such that neither $\mathbf{x} \succsim \mathbf{y}$ nor $\mathbf{y} \succsim \mathbf{x}$: some alternatives that cannot be ranked.

Definition

For any preference relation $\succsim$ on X,

- the symmetric component $\sim$ is defined by: $\mathbf{x} \sim \mathbf{y} \quad \Leftrightarrow \quad \begin{array}{c} \mathbf{x} \succsim \mathbf{y} \\ \text{and} \\ \mathbf{y} \succsim \mathbf{x} \end{array}$

- Indifference is not "incomparability": it says you can compare both ways.
- Unlike indifference, incomparability cannot usually be broken by "adding ε".
- Some of the difference could be semantic: "thick indifference curves"

Completeness and Choice

- Given a set of alternatives A, an element $\mathbf{x}$ of this set can be chosen if there is no $\mathbf{y} \in A$ that is strictly preferred to it.
- One can define the *induced choice* correspondence:

$$C_{\succ}(A) = \{\mathbf{x} \in A : \text{ there is no } \mathbf{y} \in A \text{ such that } \mathbf{y} \succ \mathbf{x}\}$$

- We call any element of $C_{\succ}(A)$ maximal.

Choice from Budget Set

Choose an allocation given prices $\mathbf{p}$ and income w:

$\mathbf{x} \in C_{\succ}(\text{Budget Set})$ if and only if

1. $\mathbf{y} \succ \mathbf{x} \Rightarrow \mathbf{p} \cdot \mathbf{y} > w$
2. $\mathbf{p} \cdot \mathbf{x} \leq w$

Completeness and Revealed Preference

- With completeness, if **x** is chosen when **y** is available one concludes **x** is (weakly) revealed preferred to **y**.
- Without completeness, if **x** is chosen when **y** is available, one can only conclude **y** is not revealed preferred to **x**.

Remark

Revealed preferences arguments need to be much more careful when one allows incompleteness.

Risk versus Uncertainty

A box contains balls of two colors. A ball will be drawn from this urn and the outcome of that draw will determine what happens.

Risk		Uncertainty	
Green	Red	Green	Red
$G = 30$	$R = 70$	$20 \leq G \leq 50$	$50 \leq G \leq 80$
$G + R = 100$		$G + R = 100$	
$P(G) = \frac{30}{100}$	$P(R) = \frac{70}{100}$	$P(G) = ?$	$P(R) = ?$

- Risk describes the special case in which one knows the probability of each event precisely.
- Uncertainty describes the more realistic case in which this is not the case.

Expected Utility without Completeness

Theorem (Bewley [1986])

Given a bunch of axioms on $\succ$, there exists a closed and convex set of probability distributions Π and a continuous function $v : X \to \mathbb{R}$ such that:

$$f \succ g \Leftrightarrow \sum_{s \in S} \pi_s u(f(s)) > \sum_{s \in S} \pi_s u(g(s)) \text{ for all } \pi \text{ in } \Pi.$$

where $u(h(s))$ is the von-Neumann & Morgenstern utility $(\sum_x h_s(x) v(x))$.

- Equivalently

$$f \succ g \quad \Leftrightarrow \quad \mathbb{E}_\pi(u \circ f) > \mathbb{E}_\pi(u \circ g) \text{ for all } \pi \text{ in } \Pi.$$

- Comparisons are done one probability distribution at a time.
- There is no utility function.
- If preferences are complete then Π is a singleton and this is expected utility.
- In Frank Knight (1921) language: completeness characterizes risk while incompleteness characterizes uncertainty (Π is not a singleton).

Expected Utility without Completeness

- Think of the simple world of state contingent amounts of money: vectors in $\mathbb{R}^S$.

Incomplete Preferences over Money

For any $\mathbf{x}, \mathbf{y} \in \mathbb{R}^S$,

$$\mathbf{x} \succ \mathbf{y} \quad \Leftrightarrow \quad \sum_{s=1}^{S} \pi_s u(\mathbf{x}_s) > \sum_{s=1}^{S} \pi_s u(\mathbf{y}_s) \text{ for all } \boldsymbol{\pi} \in \Pi$$

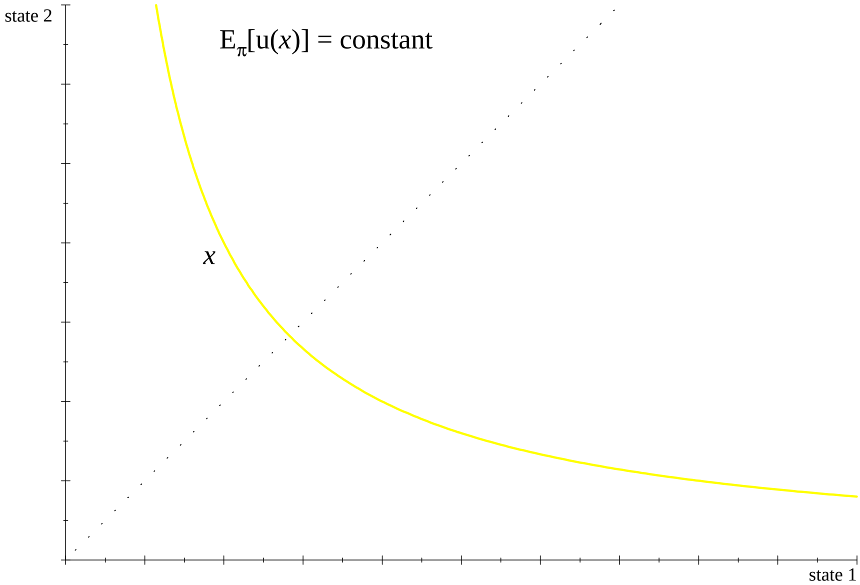
or

$$\mathbf{x} \succ \mathbf{y} \quad \Leftrightarrow \quad \mathbb{E}_{\boldsymbol{\pi}}[u(\mathbf{x})] > \mathbb{E}_{\pi}[u(\mathbf{y})] \text{ for all } \boldsymbol{\pi} \text{ in } \Pi.$$

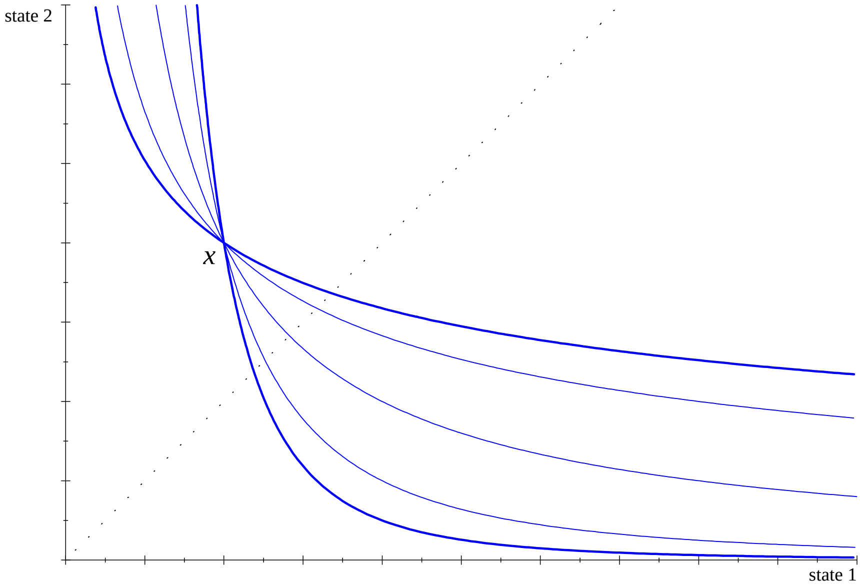
where $u(\mathbf{x}) = (u(x_1), u(x_2), ..., u(x_S))$

- Next look at how these preferences work with pictures.

Expected Utility



Bewley Preferences



Indifferent and Not Comparable are not the same

- If **x** and **y** are not comparable

$$E_{\tilde{\pi}}\left[u\left(\mathbf{x}\right)\right] > E_{\tilde{\pi}}\left[u\left(\mathbf{y}\right)\right] \text{ for some } \tilde{\pi} \text{ in } \Pi$$

  and

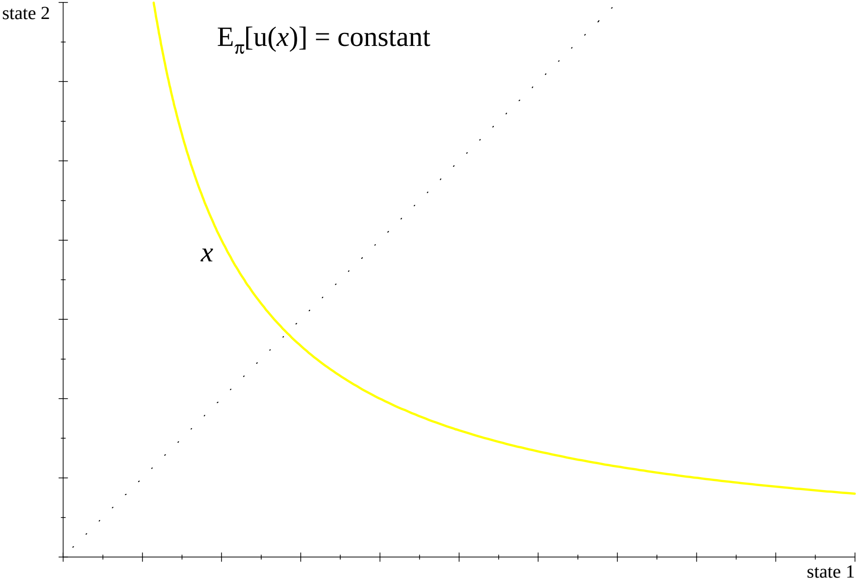
$$E_{\hat{\pi}}\left[u\left(\mathbf{x}\right)\right] < E_{\hat{\pi}}\left[u\left(\mathbf{y}\right)\right] \text{ for some } \hat{\pi} \text{ in } \Pi$$

- Indifference means

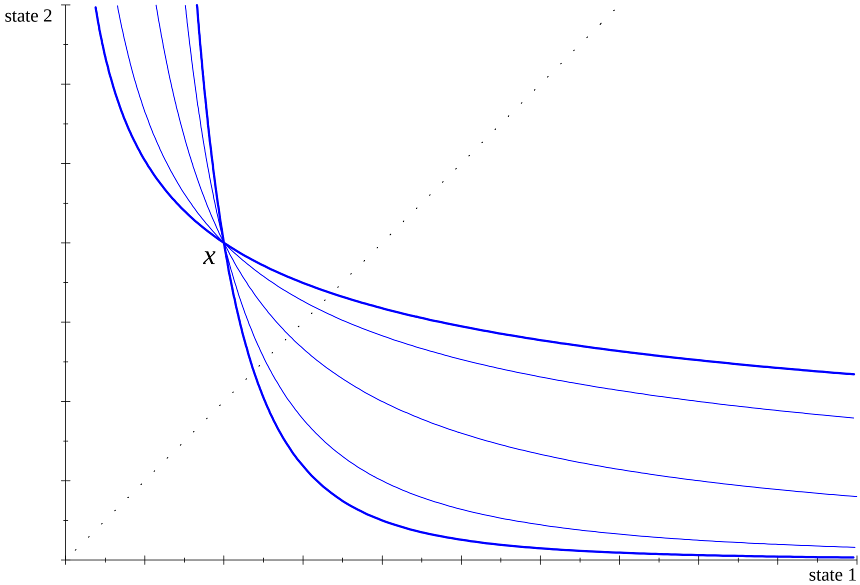
$$E_{\pi}\left[u\left(\mathbf{x}\right)\right] = E_{\pi}\left[u\left(\mathbf{y}\right)\right] \text{ for all } \pi \text{ in } \Pi$$

  which is a very restrictive condition.
- indifference can be "broken" by adding ε, incomparability cannot.

Expected Utility



Bewley Preferences



Choice from a Budget Set

- Given prices $\mathbf{p}$ and income w the Budget Set is:

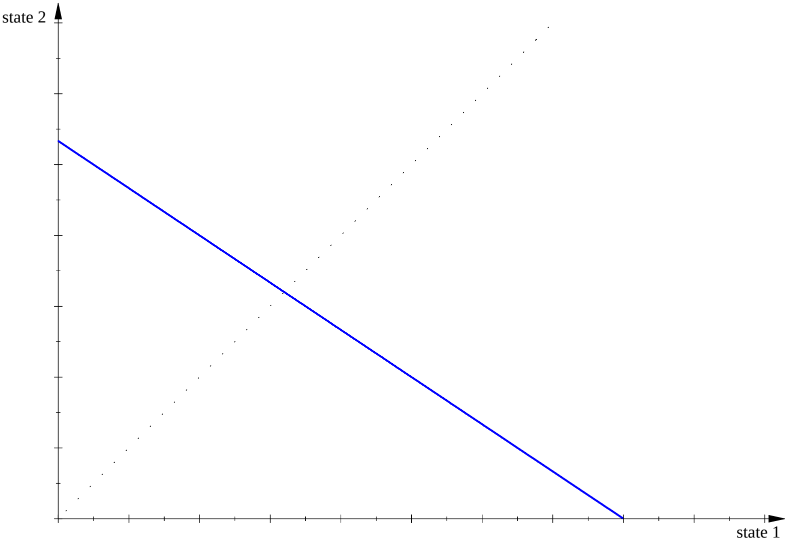
$$B(\mathbf{p}, w) = \{\mathbf{x} \in X : \mathbf{p} \cdot \mathbf{x} \leq w\}$$

- The set of maximal elements is:

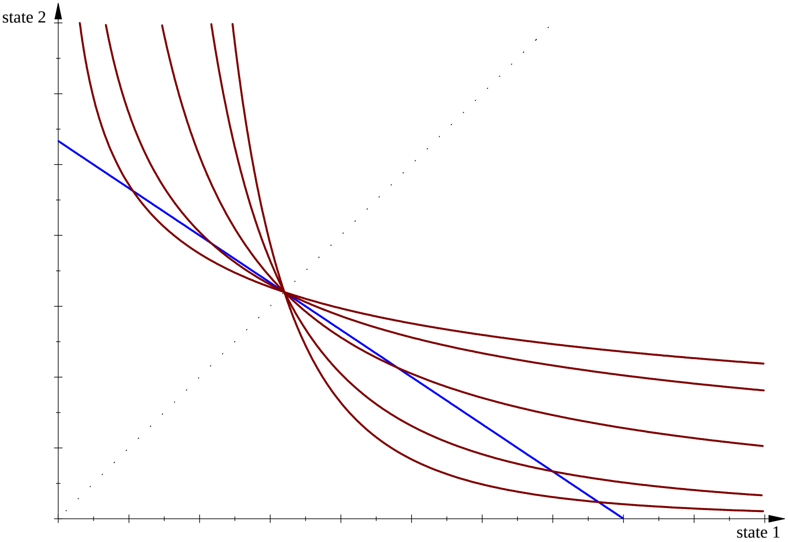
$$\{\mathbf{x} \in B : \text{ there is no } \mathbf{y} \in B \text{ such that } \mathbf{y} \succ \mathbf{x}\}$$

- Can we characterize this set?
- Let's try looking at pictures.

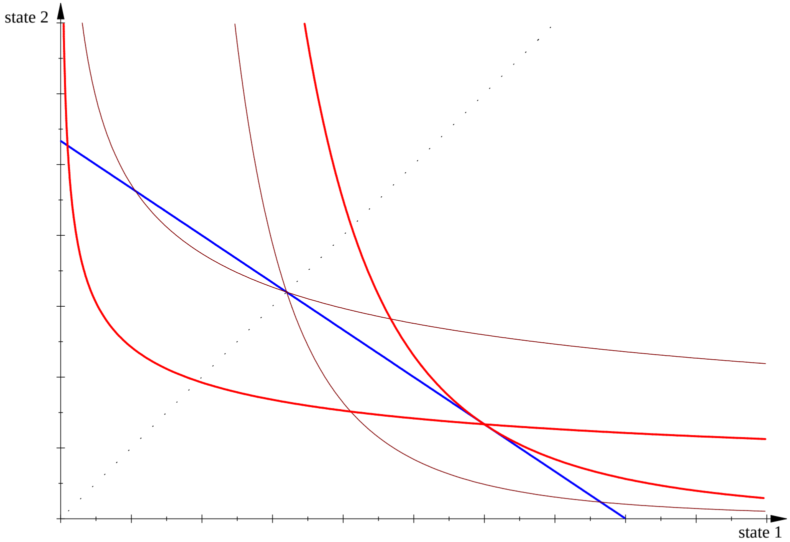
The Budget Set



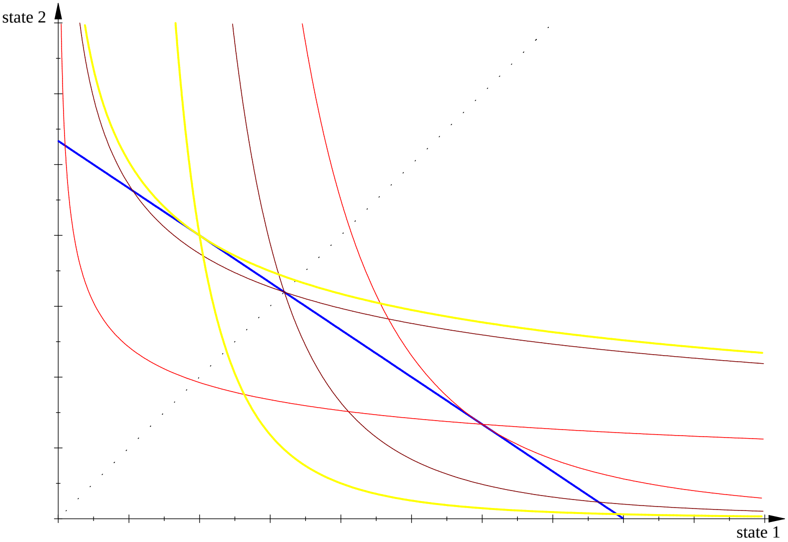
A consumption choice



Another consumption choice

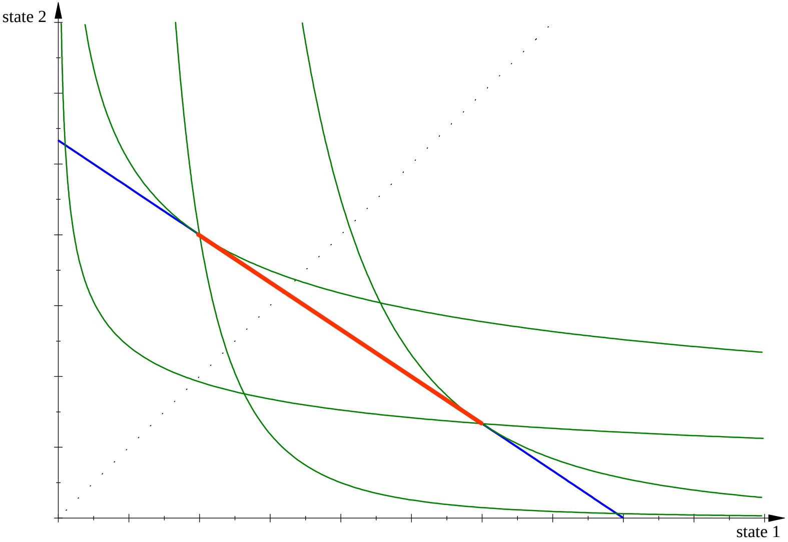


A Third consumption choice



All Maximal Consumption Bundles

Choice Is Indeterminate



How to Find Maximal Choices

- If $\mathbf{x}$ solves

$$\max_{\mathbf{x} \in \text{Budget Set}} E_{\hat{\pi}}\left[u\left(\mathbf{x}\right)\right]$$

  for some $\hat{\pi}$ in Π, then it is maximal (obvious, right?).

- The other direction also holds because the better-than set and the budget set are convex.
  - If a choice is maximal, there is a "separating hyperplane" between the better-than set at that choice and the budget set. That separating hyperplane corresponds to one element of Π.

Result

$\mathbf{x}$ is maximal if and only if $\mathbf{x}$ solves $\max_{\mathbf{x} \in \text{Budget Set}} E_{\pi}\left[u\left(\mathbf{x}\right)\right]$

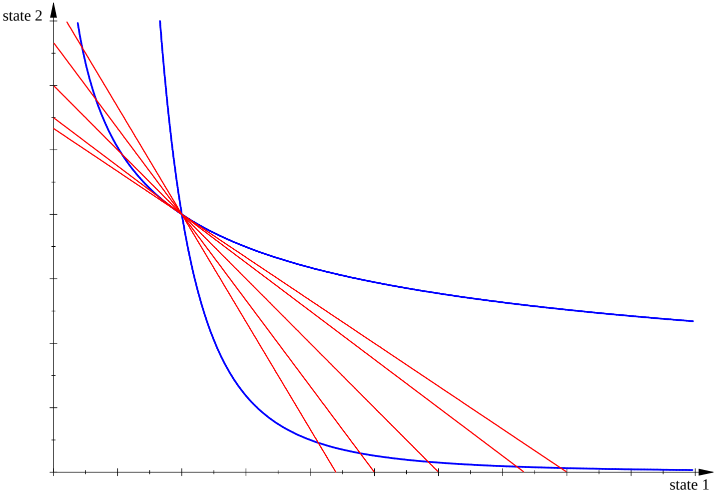
Remark

One can find all the maximal choices by picking a probability distribution and solving the corresponding expected utility maximization problem.

- This result extends to convex choice sets.

Choices Are Robust

The same consumption bundle can be chosen for different prices



- If the price vector changes a little, the same choice will remain maximal.

Wrap Up

- Without completeness one can still do a lot
- Bewley's model of decision making under uncertainty is tractable
  - Leads to interesting implications in general equilibrium theory, moral hazard, and mechanism design.
- One can take incomplete preferences to data.

References

- Bewley (1986): "KnightianDecisionTheory: Part I," Cowles Foundation Discussion Paper . Also in *Decisions in Economics and Finance*, November 2002, Volume 25, Issue 2, pp 79–110.
- Bewely (1987): "Knightian Decision Theory, Part II. Intertemporal Problems" Cowles Foundation Discussion Paper .
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- Bewley (1989): "Market Innovation and Entrepreneurship: A Knightian View". Cowles Foundation Discussion Paper.